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ORTHOTROPIC YIELD CRITERIA FOR DESCRIPTION OF THE ANISTROPY IN TENSION AND COMPRESSION OF SHEET METALS

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Orthotropic yield criteria for description of the anisotropy in tension and compression of sheet metals

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Abstract

In this paper, yield functions describing the anisotropic behavior of textured metals are proposed. These yield functions are extensions to orthotropy of the isotropic yield function proposed by Cazacu et al. (Cazacu, O., Plunkett, B., Barlat, F., 2006. Orthotropic yield criterion for hexagonal close packed metals. Int. J. Plasticity 22, 1171–1194). Anisotropy is introduced using linear transformations of the stress deviator. It is shown that the proposed anisotropic yield functions represent with great accuracy both the tensile and compressive anisotropy in yield stresses and *r*-values of materials with hcp crystal structure and of metal sheets with cubic crystal structure. Furthermore, it is demonstrated that the proposed formulations can describe very accurately the anisotropic behavior of metal sheets whose tensile and compressive stresses are equal.

It was shown that the accuracy in the description of the details of the flow and *r*-values anisotropy in both tension and compression can be further increased if more than two linear transformations are included in the formulation. If the in-plane anisotropy of the sheet in tension and compression is not very strong, the yield criterion CPB06ex2 provides a very good description of the main trends.

Keywords: Hexagonal metals; Tension/compression asymmetry; Anisotropic yield function; Linear transformations

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1. Introduction

Characterization and modeling of the anisotropy in the plastic response of metals with cubic crystal structure is well advanced. Anisotropic yield functions that capture with increased accuracy both the anisotropy of the yield strengths and the anisotropic distributions of the Lankford coefficients have been proposed (see Cazacu and Barlat, 2003; Barlat et al., 2005; Hu, 2005; etc). For metals with cubic crystal structure, the basic deformation mechanism is slip. Since slip does not depend on the sign of the shear stress i.e. can operate both forward and backward, the tensile and compressive yield stresses are equal so the yield functions are symmetric about the origin in the stress space. Metals with hexagonal close packed (hcp) crystal structure deform plastically by slip and twinning. As opposed to slip, twinning is a directional shear mechanism: shear in one direction can cause twinning while shear in the opposite direction cannot. For example, in magnesium alloys sheets twinning is not active in tension along any direction in the plane of the sheet, but is easily activated in compression. As a result the average initial in-plane compressive yield stress is about half the average in-plane tensile yield stress (e.g. see Lou et al., 2007). Thus, the yield surfaces are not symmetric with respect to the stress free condition. Since hcp metals sheets exhibit strong basal textures (c-axis oriented predominantly perpendicular to the thickness direction), a pronounced anisotropy in yielding is observed. To account for both strength differential (SD) effects and the anisotropy displayed by hcp metals, Hosford (1966) proposed the following modification of Hill's (1948) orthotropic yield criterion:

$$A\sigma_{xx} + B\sigma_{yy} + (-B - A)\sigma_{zz} + F(\sigma_{yy} - \sigma_{zz})^2 + G(\sigma_{zz} - \sigma_{xx})^2 + H(\sigma_{xx} - \sigma_{yy})^2 = 1, \qquad (1)$$

where A, B, F, G, H are material coefficients and **x**, **y**, **z** are normal to the mutually orthogonal planes of symmetry of the material . Since the criterion does not involve shear stresses, it cannot account for the continuous variation of the plastic properties between the material axes of symmetry. Liu et al. (1997) have proposed an extension of Hill (1948) yield criterion in the form:

$$\begin{aligned}&\{F(\sigma_{yy} - \sigma_{zz})^2 + G(\sigma_{zz} - \sigma_{xx})^2 + H(\sigma_{xx} - \sigma_{yy})^2 + 2L\sigma_{yz}^2 + 2M\sigma_{xz}^2 + 2N\sigma_{xy}^2\}^{1/2}\\ &\quad + I\sigma_{xx} + J\sigma_{yy} + K\sigma_{zz} = 1. \end{aligned}\qquad (2)$$

In the above equation, F, G, H, L, M, N, I, J, K are independent material coefficients. Although this yield criterion captures SD effects, the asymmetry in yielding is due to pressure effects. The effects of hydrostatic pressure on macroscopic plastic flow have been reported by Spitzig and Richmond (1984) for fully dense metals (e.g. steels). However, these effects are small at low pressure levels and lead to yield stresses which are larger in compression than in tension. For HCP metals, the predominant mechanism responsible for SD effects is twinning (Hosford and Allen, 1973), which typically leads to lower initial yield stresses in compression than in tension for in-plane loadings of rolled sheets. Therefore, the criterion in Eq. (2) may not capture with accuracy the behavior of HCP metals. Recently, based on results of polycrystalline simulations, Cazacu and Barlat (2004) have proposed a macroscopic isotropic yield criterion expressed in terms of the invariants of the stress deviator, which captures the asymmetry in yielding between tension and compression. To describe both the asymmetry and anisotropy in yielding of magnesium and magnesium alloys sheets, an extension of this criterion to orthotropy was formulated using the generalized invariants approach proposed by Cazacu and Barlat (2001). For full stress state (3D) conditions, anisotropy is described by eighteen coefficients. This orthotropic yield function is homogeneous of degree three in stresses, yet for certain hcp materials such as titanium the yield surfaces are quadratic (see Liu et al., 1997; Cazacu et al., 2006; etc.). To overcome this limitation, Cazacu et al. (2006) proposed an isotropic yield criterion for which the degree of homogeneity is not fixed. To capture simultaneously anisotropy and tension/compression asymmetry, this isotropic yield criterion was extended to orthotropy by applying a fourth-order linear operator on the stress deviator. Thus, for full 3-D stress states, nine anisotropy coefficients are involved in the criterion.

This paper presents full stress 3-D yield criteria for describing the anisotropic plastic response of textured metals. The aim is to develop models that can be applicable to materials that exhibit strength differential effects as well as to materials for which not noticeable difference exist between the behavior in tension and compression under monotonic loading. Key in this development is the use of the isotropic yield function of Cazacu et al. (2006). Anisotropy is introduced using several linear transformations. In the next section, this isotropic yield criterion is succinctly presented. After reviewing general aspects of linear transformations operating on the Cauchy stress tensor, in Section 3, a new anisotropic yield function involving two linear transformations is introduced. The input data needed for the calculation of anisotropic yield function coefficients are discussed. Illustrative examples of application of this yield function to the description of anisotropy and tension/compression asymmetry of materials with cubic and hexagonal close packed crystal structure are presented. It is shown that by incorporating more than two linear transformations in the isotropic yield criterion, a very high degree of accuracy in the representation of a very large set of anisotropic tensile and compressive data can be achieved. Moreover, it is shown that the 3-D yield criterion involving two linear transformations captures with high accuracy the anisotropy in yielding of metals with no tension/compression asymmetry.

2. Isotropic yield criterion

According to classical results of the theory of representation (e.g. Wang, 1970), the yield function of an isotropic pressure-insensitive material must be expressed in terms of the three invariants of $\boldsymbol{S}$, the deviator of stress tensor σ. Many sets of three invariants are possible (Życzkowski, 1981) but, in this work, the principal values S_k are used to represent the isotropic yield function f i.e.

$$f(S) = F(S_1, S_2, S_3), \tag{3}$$

where S_i, $i = 1 \ldots 3$ are the principal values of the stress deviator $\boldsymbol{S}$. F must be an isotropic function of its arguments, i.e.,

$$F(S_{\mathrm{I}}, S_{\mathrm{II}}, S_{\mathrm{III}}) = F(S_1, S_2, S_3),$$

where $(\mathrm{I}, \mathrm{II}, \mathrm{III})$ are permutations of $(1, 2, 3)$.

If a material only deforms by a reversible shear mechanism such as slip, yielding depends only on the magnitude of the resolved shear stress. Thus, $f(\boldsymbol{S}) = f(-\boldsymbol{S})$ and the yield locus in the deviatoric π plane (plane which passes through the origin and is perpendicular to the hydrostatic axis) must have sixfold symmetry. If the material deforms by twinning yielding depends on the sign of the applied shear stress, i.e., yield in tension and compression should be different. Hosford and Allen (1973) used a modified Taylor polycrystal model to show that for randomly oriented fcc and bcc crystals deforming solely by twinning, the ratio between the yield stress in tension and compression should be 0.78 and 1.28, respectively. To account for this strength differential effect, the following isotropic yield function was proposed by Cazacu et al. (2006)

$$\phi = (|S_1| - kS_1)^a + (|S_2| - kS_2)^a + (|S_3| - kS_3)^a, \tag{4}$$

where a and k are material parameters. It was shown that for a fixed value of the degree of homogeneity a, the parameter k is expressible solely in terms of the ratio between σ_T, the uniaxial yield in tension, and σ_C the uniaxial yield in compression, respectively, i.e.

$$k = \frac{1 - \left\{\frac{2^a - 2\cdot(\sigma_T/\sigma_C)^a}{(2\cdot\sigma_T/\sigma_C)^a - 2}\right\}^{\frac{1}{a}}}{1 + \left\{\frac{2^a - 2\cdot(\sigma_T/\sigma_C)^a}{(2\cdot\sigma_T/\sigma_C)^a - 2}\right\}^{\frac{1}{a}}}. \tag{5}$$

It was shown that this yield function is also in excellent agreement with results obtained using the self-consistent polycrystal (VPSC) model of Lebensohn and Tome (see for example Lebensohn and Tomé, 1993) for randomly oriented polycrystals deforming solely by twinning. Assuming $a = 2$, and using (5), the strength differential parameter is $k = -0.31$ for fcc materials, while $k = 0.31$ for bcc materials. Furthermore, for $a = 3$ and $k = -0.0645$, the yield loci (4) are in excellent agreement with the yield loci for randomly oriented Zr (hcp) polycrystals deforming solely by tensile twinning $\{10\bar{1}2\}\langle 10\bar{1}1\rangle$ and compressive twinning $\{11\bar{2}2\}\langle 11\bar{2}\bar{3}\rangle$ obtained using the VPSC model.

Irrespective of the value of a, if the yield stresses in tension and compression are equal $k = 0$. In particular, for $k = 0$ and $a = 2$, the yield criterion (4) reduces to the Von Mises yield criterion.

3. Anisotropic yield criteria

To extend this isotropic criterion to orthotropy, Cazacu et al. (2006) applied a linear transformation on the deviatoric stress tensor $\mathbf{S}$, i.e. in Eq. (4) S_1, S_2, S_3 were substituted by the principal values of a transformed tensor $\boldsymbol{\Sigma}$ defined as

$$\boldsymbol{\Sigma} = \mathbf{C} : \mathbf{S}. \tag{6}$$

Thus, the orthotropic yield criterion (denoted in the following as CPB06) is of the form:

$$F = \phi(\Sigma_1, \Sigma_2, \Sigma_3) = (|\Sigma_1| - k\Sigma_1)^a + (|\Sigma_2| - k\Sigma_2)^a + (|\Sigma_3| - k\Sigma_3)^a, \tag{7}$$

where Σ_1, Σ_2, Σ_3 are the principal values of Σ. The only restrictions imposed on the fourth-order tensor $\mathbf{C}$ (see Eq. (6)) are: (i) to satisfy the major and minor symmetries and (ii) to be invariant with respect to the orthotropy group. Thus, for 3-D stress conditions CPB06 involves nine independent anisotropy coefficients and it reduces to the isotropic criterion (4) when $\mathbf{C}$ is equal to the fourth-order identity tensor. It is worth noting that although the transformed tensor is not deviatoric, the orthotropic criterion is insensitive to hydrostatic pressure and thus the condition of plastic incompressibility is satisfied (see Cazacu et al., 2006 for details). For $k \in [-1, 1]$ and any integer $a \geqslant 1$, the anisotropic yield function is convex in the variables Σ_1, Σ_2, Σ_3. The experimental data on hcp materials reported in the literature (e.g. Lee and Backofen, 1966; Kelley and Hosford, 1968) is generally limited to compressive and tensile yield stresses along the axes of symmetry of the respective materials. It was shown (see Cazacu et al., 2006) that the CPB06 yield criterion describes with accuracy the tension/compression asymmetry of these materials as well as that of high-purity zirconium with in-plane isotropy (Plunkett et al., 2006). However, it is clear that nine anisotropy coefficients may not be sufficient to describe with very good accuracy yielding of materials with very pronounced in-plane anisotropy.

To increase the number of anisotropy coefficients in the formulation, instead of one linear transformation, n linear transformations ($n \geqslant 2$) can be considered. Such a methodology was used by Barlat et al. (2003, 2005, 2006) and Bron and Besson (2004) to describe the pronounced anisotropy displayed by certain aluminum alloys. For example, Yld2004-18p proposed by Barlat et al. (2005) involves two linear transformations. When the two transformations are equal, Yld2004-18p reduces to Yld91 orthotropic yield criterion (Barlat et al., 1991). Here, we demonstrate that additional linear transformations can be incorporated into the CPB06 criterion for an improved representation of the anisotropy yield surface.

The following analytic yield function, denoted CPB06ex2, is proposed:

$$\begin{aligned} F(\boldsymbol{\Sigma},\boldsymbol{\Sigma}') = {} & (|\Sigma_1| - k\Sigma_1)^a + (|\Sigma_2| - k\Sigma_2)^a + (|\Sigma_3| - k\Sigma_3)^a + (|\Sigma'_1| - k'\Sigma'_1)^a \\ & + (|\Sigma'_2| - k'\Sigma'_2)^a + (|\Sigma'_3| - k'\Sigma'_3)^a, \end{aligned} \tag{8}$$

where k and k' are material parameters that allow for the description of strength differential effects, a is the degree of homogeneity, while

$$\boldsymbol{\Sigma} = \mathbf{C} : \mathbf{S} \quad \text{and} \quad \boldsymbol{\Sigma}' = \mathbf{C}' : \mathbf{S}. \tag{9}$$

Let $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ be the reference frame associated with orthotropy. In the case of a sheet, $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ represent the rolling, transverse, and the normal directions. Relative to the orthotropy axes $(\boldsymbol{x},\boldsymbol{y},\mathbf{z})$, the fourth-order tensors $\mathbf{C}$ and $\mathbf{C}'$ operating on the stress deviator are represented by

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}, \tag{10}$$

$$C' = \begin{bmatrix} C'_{11} & C'_{12} & C'_{13} & 0 & 0 & 0 \\ C'_{12} & C'_{22} & C'_{23} & 0 & 0 & 0 \\ C'_{13} & C'_{23} & C'_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C'_{66} \end{bmatrix}.$$

Thus, for 3-D stress conditions the orthotropic criterion (8) involves 18 anisotropy coefficients. When $C_{ii} = 1$ and all other $C_{ij} = 0$ ($i \neq j$) and $k = k'$, this yield function reduces to the isotropic yield function (4). Note that when $C = C'$ and $k = k'$ the proposed criterion reduces to the CPB06 yield criterion (see Eq. (7)).

Consider uniaxial loading in the plane $(\boldsymbol{x},\boldsymbol{y})$ of the sheet along a direction at angle θ with the rolling direction and denote by $\sigma_\theta^{\mathrm{T}}$ and $\sigma_\theta^{\mathrm{C}}$ the tensile and compressive yield stresses, respectively. Then, according to the proposed criterion (8):

$$\sigma^{\mathrm{T}}(\theta)=X^{\mathrm{T}}\left\{\frac{1}{\begin{array}{l}[|A_1|-kA_1]^a+[|A_2|-kA_2]^a+[|A_3|-kA_3]^a\\ \quad+[|A_1'|-k'A_1']^a+[|A_2'|-k'A_2']^a+[|A_3'|-k'A_3']^a\end{array}}\right\}^{\frac{1}{a}}, \tag{11}$$

$$\sigma^{C}(\theta)=X^{\mathrm{T}}\left\{\frac{1}{\begin{array}{l}[|A_1|+kA_1]^a+[|A_2|+kA_2]^a+[|A_3|+kA_3]^a\\ \quad+[|A_1'|+k'A_1']^a+[|A_2'|+k'A_2']^a+[|A_3'|+k'A_3']^a\end{array}}\right\}^{\frac{1}{a}}, \tag{12}$$

where X^{T} is the tensile yield stress in the rolling direction (i.e. for $\theta=0$), and

$$\begin{aligned}
&A_1=\frac{1}{2}\left(A_{xx}+Ag+\sqrt{(A_{xx}-A_{yy})^2+4A_{xy}^2}\right),\quad A_2=\frac{1}{2}\left(A_{xx}+A_{yy}-\sqrt{(A_{xx}-A_{yy})^2+4A_{xy}^2}\right),\quad A_3=A_{zz},\\
&A_{xx}=\Phi_1\cos^2\theta+\Psi_1\sin^2\theta,\quad A_{yy}=\Phi_2\cos^2\theta+\Psi_2\sin^2\theta,\quad A_{zz}=\Phi_3\cos^2\theta+\Psi_3\sin^2\theta,\\
&A_{xy}=C_{66}\sin\theta\cos\theta,\quad \text{and}\\
&\Psi_1=\frac{2}{3}C_{12}-\frac{1}{3}C_{11}-\frac{1}{3}C_{13},\quad \Psi_2=\frac{2}{3}C_{22}-\frac{1}{3}C_{12}-\frac{1}{3}C_{23},\quad \Psi_3=\frac{2}{3}C_{23}-\frac{1}{3}C_{13}-\frac{1}{3}C_{33}.
\end{aligned} \tag{13}$$

Relations similar to (13) express Φ_1',Φ_2',Φ_3' in terms of the anisotropy coefficients C_{ij}' and A_1',A_2',A_3' as a function of the angle θ and the anisotropy coefficients C_{ij}', respectively.

In particular, X^{C}, the compressive yield stress in the rolling direction is

$$X^{\mathrm{C}}=X^{\mathrm{T}}\left\{\frac{1}{\begin{array}{l}[|\Phi_1|+k\Phi_1]^a+[|\Phi_2|+k\Phi_2]^a+[|\Phi_3|+k\Phi_3]^a\\ \quad+[|\Phi_1'|+k'\Phi_1']^a+[|\Phi_2'|+k'\Phi_2']^a+[|\Phi_3'|+k'\Phi_3']^a\end{array}}\right\}^{\frac{1}{a}}, \tag{14}$$

where

$$\Phi_1=\frac{2}{3}C_{11}-\frac{1}{3}C_{12}-\frac{1}{3}C_{13},\quad \Phi_2=\frac{2}{3}C_{12}-\frac{1}{3}C_{22}-\frac{1}{3}C_{23},\quad \Phi_3=\frac{2}{3}C_{13}-\frac{1}{3}C_{23}-\frac{1}{3}C_{33}. \tag{15}$$

If Y^{T} and Y^{C} are the tensile and compressive yield stresses in the ***y***-direction (transverse direction) then

$$Y^{\mathrm{T}}=X^{\mathrm{T}}\left\{\frac{1}{\begin{array}{l}[|\Psi_1|-k\Psi_1]^a+[|\Psi_2|-k\Psi_2]^a+[|\Psi_3|-k\Psi_3]^a\\ \quad+[|\Psi_1'|-k'\Psi_1']^a+[|\Psi_2'|-k'\Psi_2']^a+[|\Psi_3'|-k'\Psi_3']^a\end{array}}\right\}^{\frac{1}{a}}, \tag{16}$$

$$Y^{\mathrm{C}}=X^{\mathrm{T}}\left\{\frac{1}{\begin{array}{l}[|\Psi_1|+k\Psi_1]^a+[|\Psi_2|+k\Psi_2]^a+[|\Psi_3|+k\Psi_3]^a\\ \quad+[|\Psi_1'|+k'\Psi_1']^a+[|\Psi_2'|+k'\Psi_2']^a+[|\Psi_3'|+k'\Psi_3']^a\end{array}}\right\}^{\frac{1}{a}}. \tag{17}$$

If Z^{T} and Z^{C} are the tensile and compressive yield stresses in the ***z***-direction (normal to the sheet) then

$$Z^{\mathrm{T}} = X^{\mathrm{T}} \left\{ \frac{1}{\begin{array}{l}[|\Pi_1| - k\Pi_1]^a + [|\Pi_2| - k\Pi_2]^a + [|\Pi_3| - k\Pi_3]^a \\ \quad +[|\Pi'_1| - k'\Pi'_1]^a + [|\Pi'_2| - k'\Pi'_2]^a + [|\Pi'_3| - k'\Pi'_3]^a\end{array}} \right\}^{\frac{1}{a}}, \tag{18}$$

$$Z^{\mathrm{C}} = X^{\mathrm{T}} \left\{ \frac{1}{\begin{array}{l}[|\Pi_1| + k\Pi_1]^a + [|\Pi_2| + k\Pi_2]^a + [|\Pi_3| + k\Pi_3]^a \\ \quad +[|\Pi'_1| + k'\Pi'_1]^a + [|\Pi'_2| + k'\Pi'_2]^a + [|\Pi'_3| + k'\Pi'_3]^a\end{array}} \right\}^{\frac{1}{a}}, \tag{19}$$

where

$$\Pi_1 = \frac{2}{3}C_{13} - \frac{1}{3}C_{11} - \frac{1}{3}C_{12}, \quad \Pi_2 = \frac{2}{3}C_{23} - \frac{1}{3}C_{12} - \frac{1}{3}C_{22}, \quad \Pi_3 = \frac{2}{3}C_{33} - \frac{1}{3}C_{13} - \frac{1}{3}C_{23} \tag{20}$$

and relations similar to (20) express each Π' term as a function the anisotropy coefficients C'_{ij}. Due to plastic incompressibility, yielding under equibiaxial tension and compression occurs when σ_{xx} and σ_{yy} are both equal to Z^{C} and Z^{T}, respectively.

The yield stress in pure shear in the sheet plane is

$$\tau_{xy} = X^{\mathrm{T}} \left\{ \frac{1}{[|C_{66}| + kC_{66}]^a + [|C_{66}| - kC_{66}]^a + [|C'_{66}| + kC'_{66}]^a + [|C'_{66}| - k'C'_{661}]^a} \right\}^{\frac{1}{a}} \tag{21}$$

and the yield stresses in shear with respect to the other directions of orthotropy can be found by the substitution of C_{44}, and C'_{44} or C_{55}, and C'_{55} in the place of C_{66}, and C'_{66}.

Furthermore, we assume that the plastic potential coincides with the yield function. Let denote by r_θ the Lankford coefficient (width to thickness strain ratios) under uniaxial tensile or compressive loading in a direction at angle θ with the rolling direction in the plane (xy). Hence

$$r_\theta = -\frac{\sin^2\theta \frac{\partial F}{\partial \sigma_{xx}} - \sin(2\theta)\frac{\partial F}{\partial \sigma_{xy}} + \cos^2\theta \frac{\partial F}{\partial \sigma_{yy}}}{\frac{\partial F}{\partial \sigma_{xx}} + \frac{\partial F}{\partial \sigma_{yy}}}, \tag{22}$$

where $F(\Sigma, \Sigma')$ is given by Eq. (8) and the explicit expressions of the derivatives in terms of the components of the Cauchy stress are given in Appendix A.

Similarly, more linear transformations can be incorporated in the orthotropic yield criterion (8) (denoted CPB06ex3 for 3 transformations, etc.) until a desired level of accuracy in the description of the plastic anisotropy is obtained, i.e.

$$F(\mathbf{S}) = f^{(1)} + f^{(2)} + \cdots + f^{(p)} + f^{(n)}, \quad \text{with } 1 \leqslant p \leqslant n, \tag{23}$$

where $f^{(p)} = (|\Sigma_1^{(p)}| - k^{(p)}\Sigma_1^{(p)})^a + (|\Sigma_2^{(p)}| - k^{(p)}\Sigma_2^{(p)})^{\mathrm{a}} + (|\Sigma_3^{(p)}| - k^{(p)}\Sigma_3^{(p)})^{\mathrm{a}}$. In Eq. (23) $k^{(\mathrm{p})}$ are material parameters, $\Sigma_i^{(p)}, i = 1 \ldots 3$ are the principal values of $\mathbf{\Sigma}^{(p)} = \mathbf{C}^{(p)}\mathbf{S}$ with $\mathbf{C}^{(p)}$ being the fourth-order orthotropic tensor associated to the pth linear transformation.

The anisotropy coefficients involved in the yield criteria can be found through the minimization of an error function. The experimental data in the error function may consist of flow stresses and r-values in tension and compression corresponding to different orientations in the plane of the sheet, biaxial flow stress in tension and compression, as well as out-of-plane yield stresses and r-values. If experimental data are not available for a given strain path, they can be substituted with numerical data obtained based on polycrystalline calculations

$$E(C', C'', \ldots, C^{(p)}) = \sum_i w_i \left(\frac{\sigma_i^{\text{th}}}{\sigma_i^{\text{exp}}} - 1 \right)^2 + \sum_j w_j \left(\frac{r_j^{\text{th}}}{r_j^{\text{exp}}} - 1 \right)^2. \tag{24}$$

In the above equation i represents the number of experimental yield stresses, j represents the number of experimental r-values available while the superscript indicates whether the corresponding value is experimental or predicted.

In the next section, we will apply the CPB06ex2 CPB06ex3, and CPB06ex4 formulations to the description of the anisotropy and tension compression asymmetry of HCP, BCC, and FCC alloys.

4. Application to materials exhibiting asymmetry between tension and compression

4.1. AZ31B magnesium (HCP)

Lou et al. (2007), reported results of an experimental investigation of the monotonic and cyclic mechanical behavior of a commercial AZ31B magnesium alloy sheet in tension, compression, and simple shear. The characterization of the in-plane anisotropy of the sheet was done by performing uniaxial tensile and compression tests in the rolling direction, at 45° to the rolling direction and in the transverse direction. The material displays significant anisotropy as well as asymmetric yield and hardening behavior which is associated with the polarity of twinning (the easily activated twin $\{10\bar{1}2\}$ is a tensile twin). In this paper, we are only concerned with modeling with accuracy the initial yielding of the material. The data reported at 0.2% offset are sufficient for constructing the tension–tension and compression–compression quadrants of the experimental plane-stress yield locus. We will now compare Lou et al. (2007) to the theoretical yield locus calculated with the proposed orthotropic yield criterion CPB06ex2 (Eq. (8)) in Fig. 1. The parameters involved in the expression of the theoretical yield loci for in-plane (xy) stress states were determined using equations (1.11) and (1.12) and all the available data. For this material, $a = 4$. The values of these anisotropy coefficients are given in Table 1. Fig. 2 compares experimental and calculated yield stresses normalized by the uniaxial tensile stress in the rolling direction (0.2% offset) and r-values as a function of the loading direction (angle from the RD). Since all available experimental data were used to identify the coefficients involved in the criterion, the calculations are not really predictive. However, it is clearly seen that the CPB06ex2 yield criterion is flexible enough to describe the whole set of anisotropic data (12 independent material properties, i.e. both r-values and yield stresses) for both tensile and compressive loadings. This flexibility cannot be achieved with models that are based on one linear transformation.

4.2. Medium carbon low alloy steel (BCC)

The second application example of CPB06ex2 yield criterion (see Eq. (8)) pertains to a medium carbon alloy steel plate for which Benzerga et al. (2004) reported asymmetry between the yield behavior in tension and compression. The anisotropy of deformation was characterized through a series of tension and compression tests along the rolling (x), transverse (y), and normal direction (z) as well as along 45° orientations in the (xy),

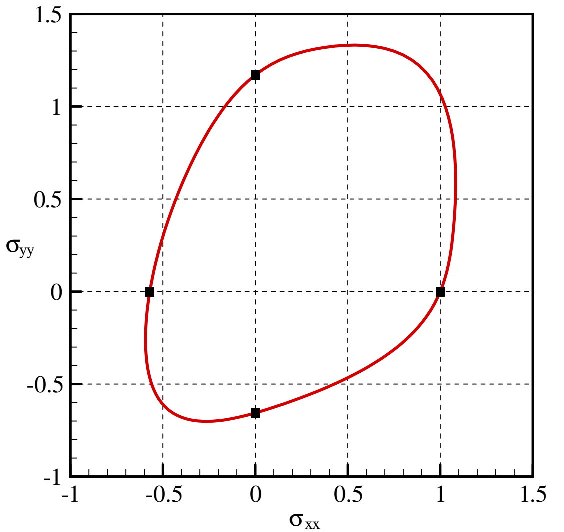

Fig. 1. Plane-stress yield loci ($\sigma_{xy} = 0$) for AZ31B Mg alloy according with the proposed CPB06ex2 yield criterion and experiments (symbols). Data after Lou et al. (2007).

(yz), and (zx) planes, respectively. Under off-axes loading in any symmetry plane, the strain ratios were defined as the in-plane radial strain over the out-of-plane radial strain. For this BCC crystal structure material, it was observed that in compression there is no significant difference between the yield stresses along the x, y, and z orientations although the corresponding r-values are different (see Tables 3 and 4). All the available experimental data (14 measured yield stress and r-values) were used to determine the coefficients of the CP05ex2 yield criterion. For this material, $a = 5$. The values of these anisotropy coefficients are given in Table 2.

Fig. 3 shows the theoretical yield surface in the normalized stress plane (σ_x, σ_y) along with the experimental data points. In Tables 3 and 4 are given the values of the calculated and measured yield stresses and Lankford coefficients. Note that the proposed theory captures very well the observed asymmetry in yielding between tension and compression, the strong anisotropy in r-values and a relatively mild anisotropy in yield stresses for both tension and compression. This flexibility cannot be achieved with Hill's (1948) yield criterion which is generally used for modeling steels.

4.3. 2090-T3 aluminum (FCC)

It was reported by Barlat et al. (1991) that for a cold rolled 2090-T3 aluminum alloy sheet under biaxial plane-stress loading conditions, the tension–tension (I) and compres-

Table 1
CPB06ex2 yield criterion parameters for AZ31B Mg alloy

k	C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{66}
0.295	0.771	−0.914	−1.312	0.654	−1.209	−0.293	1.617
k'	C'_{11}	C'_{12}	C'_{13}	C'_{22}	C'_{23}	C'_{33}	C'_{66}
0.900	0.771	0.337	−0.764	0.646	−0.401	−0.022	0.499

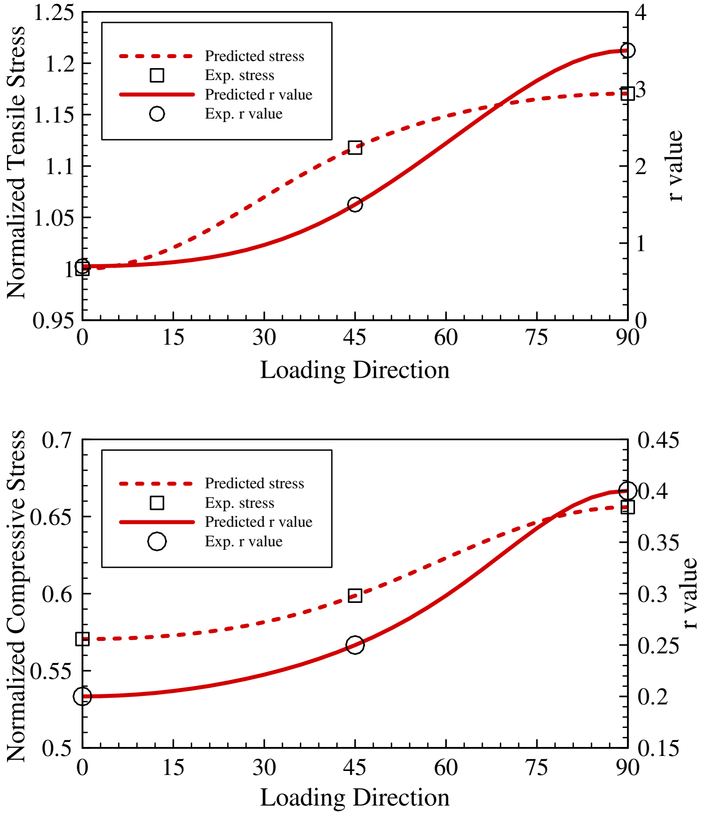

Fig. 2. Anisotropy of the yield stress (normalized by the tensile stress in the rolling direction) and the r-values for AZ31B Mg, measured and predicted with the CPB06ex2 yield criterion for (a) tensile loading; (b) compressive loading. Experimental data after Lou et al. (2007).

sion–compression quadrants (IV) are not identical due to a yielding difference in tension and compression. However, this SD effect is a snapshot of the material behavior at yield due to micro-plasticity effects related to prior thermo-mechanical processing. Aluminum alloys deformed beyond 1–2% are expected to behave the same (identical stress–strain curves) in tension and compression. The deformation mechanisms in tension and compression are not fundamentally different in aluminum alloys, in contrast with HCP materials. This example is provided in order to show the capability of the model to capture simultaneously the SD as well as strong anisotropy effects for a material that was characterized extensively, i.e., with a large number of experimental data to account for.

Table 2
CPB06ex2 coefficients for a medium carbon low alloy steel

k	C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{44}	C_{55}	C_{66}
0.030	1.283	0.208	−0.421	0.535	−0.791	0.746	1.418	1.329	1.156
k'	C'_{11}	C'_{12}	C'_{13}	C'_{22}	C'_{23}	C'_{33}	C'_{44}	C'_{55}	C'_{66}
−0.027	1.283	0.380	−0.001	2.145	0.792	0.218	1.418	1.329	1.721

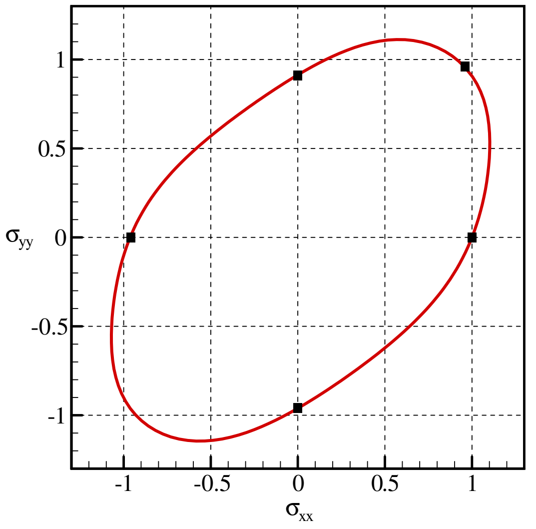

Fig. 3. Projection of the CPB06ex2 yield surface in the (σ_x, σ_y) plane for a medium carbon low alloy steel. Symbols represent normalized experimental data of Benzerga et al. (2004).

Table 3
Measured and theoretical normalized yield stress values for a medium carbon low alloy steel. Experimental data after Benzerga et al. (2004)

	Tension		Compression		
	X_T	Y_T	X_C	Y_C	Z_C
Experiment	1	0.914	0.963	0.960	0.958
CPB06ex2	1	0.914	0.963	0.960	0.958

Table 4
Measured and predicted r-values for a medium carbon low alloy steel

Loading direction	Tension		Compression					
	x	y	x	y	z	45°in (xy) plane	45°in (xz) plane	45°in (yz) plane
Experiment	0.67	1.4	0.71	1.3	1.05	0.91	1.45	1.3
CPB06ex2	0.67	1.4	0.71	1.3	1.05	0.91	1.45	1.3

Experimental data after Benzerga et al. (2004).

The experimental data consists of tensile and compressive yield stresses measured along seven directions in the plane of the sheet (i.e. from rolling to transverse directions in 15° increments). The strain-ratio anisotropy was measured only for tensile conditions. Biaxial flow stress from a bulge test, and the biaxial r value from the disk compression test (r_b) were reported in Barlat et al. (2003). The description of the uniaxial tension properties of this material to a desired degree of accuracy has posed challenges. Many anisotropic yield functions of different levels of complexity have been proposed in order to capture both the anisotropy in the tensile flow stresses and tensile r-values. However, with the exception of the yield function proposed by Bron and Besson (2004) and Barlat et al. (2005) most of the formulations were developed only for plane-stress states or do not fulfill automatically the requirements for convexity. To describe the anisotropic tensile properties of the material and at the same time account for the difference in flow stresses between tension and compression in the rolling and transverse directions, Yoon et al. (2000) used the anisotropic yield function Yld96 (Barlat et al., 1997) and kinematic hardening. However, with this approach, the accuracy in representing the tensile and compressive flow stresses for off-axes loading in the plane of the sheet (i.e. in directions other than the RD and TD) is reduced. In this paper, we will show that by introducing several linear transformations in the isotropic yield criterion (4) (denoted CPB06ex3 for three linear transformations, CPB06ex4 for four linear transformations) the desired level of accuracy in the description of the plastic anisotropy for both tensile flow stresses, compressive flow stresses and tensile r-values can be achieved. Specifically, we model the material using the proposed yield criterion with 2, 3 or 4 transformations, respectively. For the determination of the material parameters involved in each yield criterion we use all the available experimental data (a total of 22 data points). For this material, the exponent $a = 12$. A high value for the homogeneity exponent was necessary to capture with accuracy the material's response. The value of the exponent "a" was adjusted until the best approximation of the entire data set was obtained. For each yield criterion CPB06ex2, CPB06ex3, CPB06ex4 and the values of the coefficients associated with strength differential effects are listed in Tables 5–7, respectively.

Fig. 4 shows the plane-stress yield loci ($\tau_{xy} = 0$) calculated using CPB06ex2 (14 anisotropy coefficients for plane stress), CPB06ex3 (21 anisotropy coefficients for plane stress) and CPB06ex4 (28 anisotropy coefficients for plane-stress states) along with the experimental flow stresses. Fig. 5 shows the anisotropy of the tensile and compressive flow stresses and tensile r-values as described by CPB06ex2, CPB06ex3, and CPB06ex4 in comparison with the experimental values. It can be noted that although the tension–compression anisotropy of the yield loci is captured with accuracy by all the anisotropic yield criteria (see Fig. 4), it is clearly seen that as more linear transformations are involved in the

Table 5
CPB06ex2 coefficients for 2090-T3

k	C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{66}
0.054	0.453	−0.841	−1.248	−1.058	−2.284	−3.201	1.026
k'	C'_{11}	C'_{12}	C'_{13}	C'_{22}	C'_{23}	C'_{33}	C'_{66}
0.027	0.453	−0.705	1.148	0.139	−0.519	0.878	1.978

Table 6
CPB06ex3 coefficients for 2090-T3

k	C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{66}
−0.076	4.192	5.311	4.031	4.488	5.443	5.671	2.074
k'	C'_{11}	C'_{12}	C'_{13}	C'_{22}	C'_{23}	C'_{33}	C'_{66}
−0.020	4.192	5.095	6.135	6.750	5.421	5.696	1.000
k''	C''_{11}	C''_{12}	C''_{13}	C''_{22}	C''_{23}	C''_{33}	C''_{66}
−0.029	4.192	3.412	2.562	4.629	2.794	3.205	1.422

Table 7
CPB06ex4 coefficients for 2090-T3

k	C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{66}
0.331	0.417	−0.709	−0.588	0.277	−1.053	−1.949	1.275
k'	C'_{11}	C'_{12}	C'_{13}	C'_{22}	C'_{23}	C'_{33}	C'_{66}
−0.311	0.417	−0.442	−0.549	−0.975	−1.793	−0.924	−0.820
k''	C''_{11}	C''_{12}	C''_{13}	C''_{22}	C''_{23}	C''_{33}	C''_{66}
0.063	0.417	0.214	1.295	−0.057	0.065	−0.624	1.924
k'''	C'''_{11}	C'''_{12}	C'''_{13}	C'''_{22}	C'''_{23}	C'''_{33}	C'''_{66}
0.625	0.417	−0.297	0.856	−0.570	−0.160	0.775	0.938

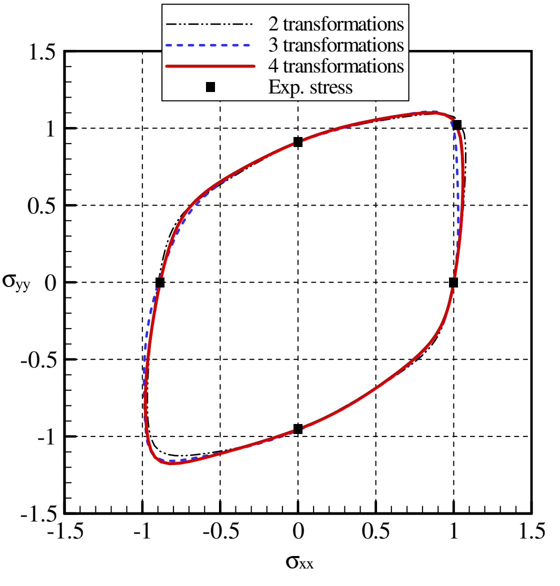

Fig. 4. Comparison between experimental and theoretical plane-stress yield surfaced according to the yield criteria CPB06ex2, CPB06ex3, and CPB06ex4 for 2090-T3 aluminum alloy sheet (data after Yoon et al., 2000).

formulation (i.e. as the number of anisotropy coefficients is increased) the accuracy is significantly improved. When four linear transformations are introduced, the error associated with representing all 22 experimental data points tends towards zero.

5. Application of the CPB06ex2 yield criterion to textured metal sheets exhibiting symmetry between tensile and compressive properties

Although the CPB06 yield criterion (Cazacu et al., 2006) and the extensions proposed in this paper were formulated to capture the strength differential effects most often associated with materials that display tension/compression asymmetry in yielding, the proposed formulations are not limited to such materials. If the yield in tension is equal to the yield in compression, the parameters k and k' associated with strength differential effects are automatically zero.

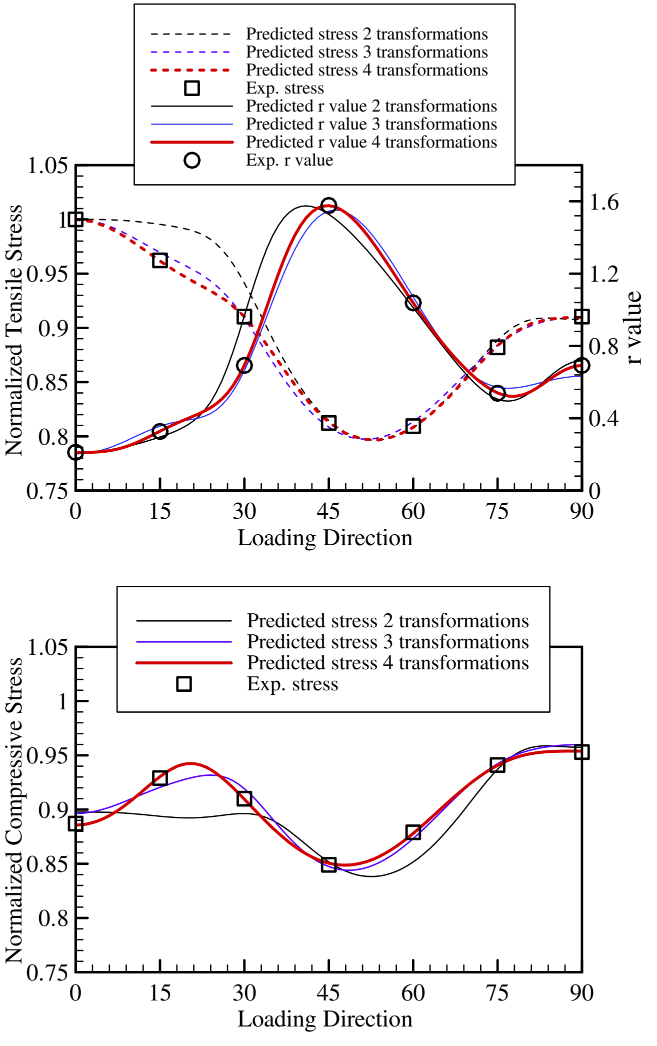

Fig. 5. Anisotropy of both tensile and compressive flow stresses (normalized by uniaxial tension in the rolling direction) and tensile r-values for 2090-T3 alloy sheet, measured (data after Yoon et al., 2000) and predicted by the yield criteria CPB06ex2, CPB06ex3 and CPB06ex4.

In this section, we apply the proposed CPB06ex2 yield criterion (Eq. (8)) to materials with cubic crystal structure (FCC and BCC) for which it is commonly assumed that the tensile and compressive yield stresses are equal. The first example pertains to 6111-T4 alu-

Table 8
CPB06ex2 yield criterion parameters for 6111-T4 Al alloy

C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{44}	C_{55}	C_{66}
0.250	−1.399	−0.971	−0.375	−2.254	−0.831	1.403	1.101	1.316
C'_{11}	C'_{12}	C'_{13}	C'_{22}	C'_{23}	C'_{33}	C'_{44}	C'_{55}	C'_{66}
0.250	−0.840	−1.330	0.780	−0.565	0.479	1.094	1.448	1.413

minum alloy sheet sample (data after Barlat et al., 2005). Tensile yield stresses and r-values along seven directions in the plane of the sheet (i.e. from rolling to transverse directions in 15° increments) were measured from uniaxial tensile tests. Additionally, the experimental values of the balanced biaxial yield stress as well as the corresponding r-value obtained using a disk compression test were reported. In addition to the experimental data, VPSC polycrystal calculations were carried out to determine uniaxial yield stresses at 45° and the yield stresses in simple shear in the Y–Z and X–Z planes. For this material the exponent $a = 12$, while the values of the normalized anisotropy coefficients are given in Table 8. Table 9 lists the reported VPSC yield stresses and corresponding CPB06ex2 predictions. Figs. 6 and 7 compare the CPB06ex2 plane-stress yield locus ($\sigma_{xy} = 0$) and the predicted uniaxial flow properties with the experimental data, respectively. The measured r_b-value is 1.225, while the CPB06ex2 yield criterion prediction is of 1.227. The very good agreement shows that CPB06ex2 is flexible enough to describe the whole set of anisotropic data (r-values and yield stresses) simultaneously.

In the previous section, we modeled the strongly textured 2090-T3 aluminum alloy sample taking into consideration the observed asymmetry between tension and compression of this material. However, in the literature, this tension/compression asymmetry is generally neglected. Next, we assume that the yield surface of the material is symmetric about the origin in the stress space and apply the yield criterion CPB06ex2 with $k = 0$ and $k' = 0$. The values of the coefficients of anisotropy for this material are given in Table 10

Table 9
Comparison between theoretical normalized yield stress values for 6111-T4 Al alloy

	Y–Z 45° tension	Z–X 45° tension	Y–Z simple shear	Z–X simple shear
VPSC	1.07	1.04	0.67	0.65
CPB06ex2	1.07	1.04	0.67	0.65

VPSC results after Barlat et al. (2005).

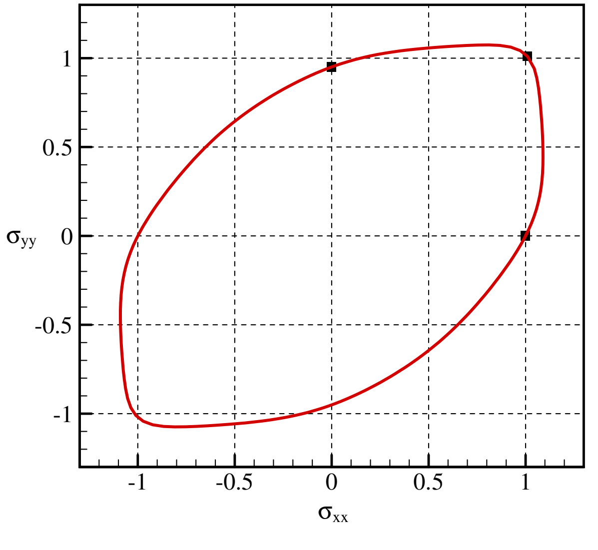

Fig. 6. CPB06ex2 ($k = 0, k' = 0$) yield surfaces for plane stress in the X–Y plane for 6111-T4 aluminum.

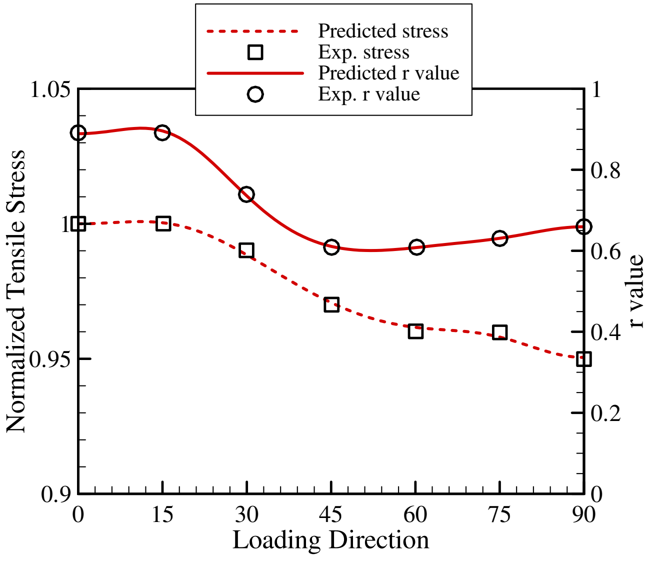

Fig. 7. Anisotropy of flow stresses (normalized by uniaxial tension about the x-axis) and r-values for 6111-T4, measured and predicted by CPB06ex2 ($k = 0, k' = 0$).

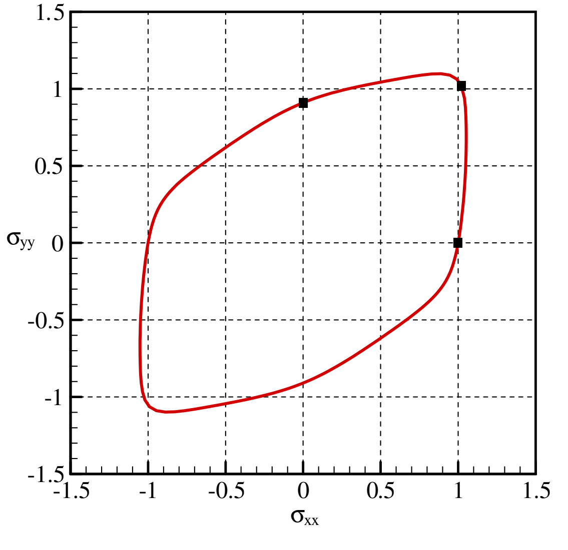

Fig. 8. Plane-stress yield surface ($\sigma_{xy} = 0$) for 2090-T3 aluminum sheet according to the CPB06ex2 yield criterion ($k = 0, k' = 0$) yield surface and experimental data (tension only). Data after Barlat et al. (2005).

($a = 12$). Furthermore, a comparison between the 45° yield stress values predicted using CPB06ex2 and the VPSC values reported by Barlat et al. (2005) is given in Table 11. Figs. 8 and 9 illustrate the ability of the model to represent the material with a high degree of accuracy. The variation in r-values with 6 peaks between 0° and 360° indicates that this model is likely to be suitable for the prediction of six or more ears in cups drawn from circular blanks.

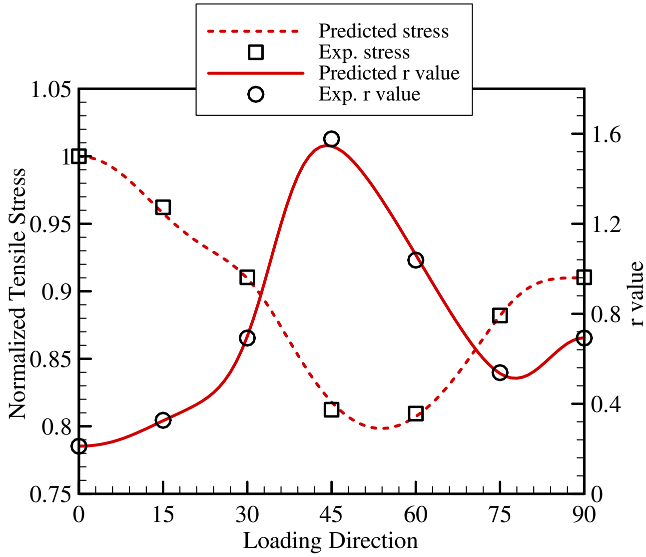

Fig. 9. Anisotropy of flow stresses (normalized by uniaxial yield stress along the rolling direction) and r-values for 2090-T3, measured and calculated with CPB06ex2 yield criterion ($k = 0, k' = 0$). Data after Barlat et al. (2005).

Table 10
CPB06ex2 yield criterion parameters for 2090-T3 Al alloy (tension only)

C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{44}	C_{55}	C_{66}
0.476	−0.724	−1.209	−0.987	−2.098	−3.070	0.563	2.008	1.117
C'_{11}	C'_{12}	C'_{13}	C'_{22}	C'_{23}	C'_{33}	C'_{44}	C'_{55}	C'_{66}
0.476	−0.763	1.143	0.095	−0.608	0.715	1.940	1.032	2.028

Table 11
Comparison between theoretical normalized yield stress values for 2090-T3 Al alloy

	Y–Z 45° Tension	Z–X 45° Tension	Y–Z simple shear	Z–X simple shear
VPSC	0.90	0.89	0.47	0.47
CPB06ex2	0.9	0.89	0.49	0.47

VPSC results after Barlat et al. (2005).

The last example pertains to Y350 Mpa cold rolled high strength steel (HSS). The experimental data consisting of tensile yield stresses and r-values along seven directions in the plane of the sheet (i.e. from rolling to transverse directions in 15° increments) were reported in Hu (2005). For this material the exponent $a = 5$. The values of the coefficients of anisotropy coefficients involved in the CPB06ex2 yield criterion are given in Table 12. Fig. 10 shows the plane-stress yield surface calculated with CPB06ex2 using the experimental yield stresses. Fig. 11 compares the experimental and predicted anisotropic properties of the steel sample. An excellent agreement with the experimental data is obtained.

Table 12
CPB06ex2 yield criterion parameters for Y350 MPa steel

C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{66}
1.761	0.630	0.475	1.712	0.229	1.664	1.499
C'_{11}	C'_{12}	C'_{13}	C'_{22}	C'_{23}	C'_{33}	C'_{66}
1.761	2.913	2.661	4.616	3.358	4.352	0.761

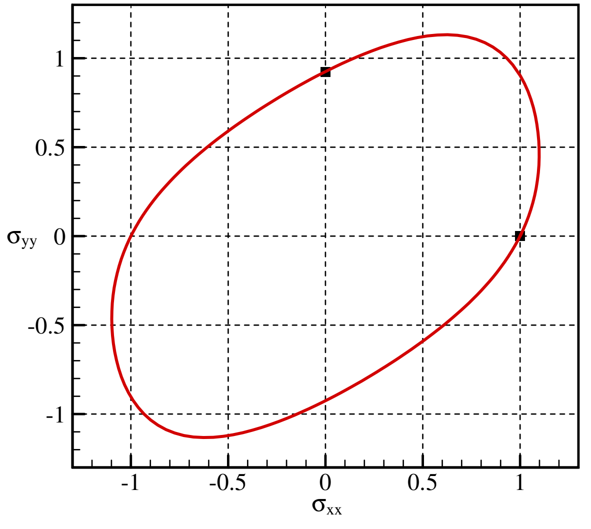

Fig. 10. The CPB06ex2 ($k = 0, k' = 0$) yield surface for plane stress in the X–Y plane for HSS steel. Experimental data from Hu (2005).

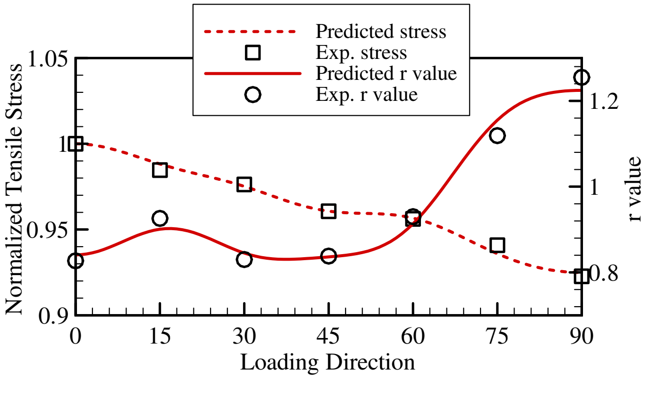

Fig. 11. Anisotropy of flow stresses (normalized by uniaxial tension about the x-axis) and r-values for HSS steel measured and predicted by CPB06ex2 ($k = 0$). Data after Hu (2005).

6. Conclusions

In this paper, yield functions describing the anisotropic behavior of textured metals were proposed. These yield functions are extensions to orthotropy of the isotropic yield function proposed by Cazacu et al. (2006). Anisotropy is introduced using linear transformations of the stress deviator. It is shown that the anisotropic yield function CPB06ex2 which involves 18 anisotropy coefficients (two linear transformations) represents with great accuracy both the tensile and compressive anisotropy in yield stresses and r-values of materials with hcp crystal structure and of metal sheets with cubic crystal structure. Furthermore, it was demonstrated that it can describe very accurately the anisotropic behavior of metal sheets whose tensile and compressive stresses are equal.

It was shown that the accuracy in the description of the details of the flow stress and r-values anisotropy in both tension and compression can be further increased if more than two linear transformations are included in the formulation. If the in-plane anisotropy of the sheet is tension and compression is not very strong, the yield criterion CPB06ex2 provides a very good description of the main trends.

Appendix A. Derivatives of the CPB06ex2 function

In the current work, we assume an associated flow rule. Let denote by r_θ the Lankford coefficient (width to thickness strain ratios) under uniaxial tensile or compressive loading in a direction at angle θ with the rolling direction in the plane (xy). Hence:

$$r_\theta = -\frac{\sin^2\theta\frac{\partial F}{\partial\sigma_{xx}} - \sin(2\theta)\frac{\partial F}{\partial\sigma_{xy}} + \cos^2\theta\frac{\partial F}{\partial\sigma_{yy}}}{\frac{\partial F}{\partial\sigma_{xx}} + \frac{\partial F}{\partial\sigma_{yy}}}, \tag{A.1}$$

where $F(\Sigma, \Sigma')$ is given by Eq. (8). The derivatives of F reduce to the following form for uniaxial tension in the x–y plane, and are required for the calculation of r_θ:

$$\frac{\partial F}{\partial\sigma_{ij}} = \frac{\partial F}{\partial\Sigma_m}\frac{\partial\Sigma_m}{\partial\Sigma_{kl}}\frac{\partial\Sigma_{kl}}{\partial\sigma_{ij}} + \frac{\partial F}{\partial\Sigma'_m}\frac{\partial\Sigma'_m}{\partial\Sigma'_{kl}}\frac{\partial\Sigma'_{kl}}{\partial\sigma_{ij}} \tag{A.2}$$

The non-zero terms in (A.2) are given by

$$\begin{aligned}
&\frac{\partial F}{\partial\Sigma_m} = a(|\Sigma_m| - k)^{a-1}\left(\frac{|\Sigma_m|}{\Sigma_m} - k\right),\\
&\frac{\partial\Sigma_1}{\partial\Sigma_{xx}} = \frac{1}{2} + \frac{\Sigma_{xx} - \Sigma_{yy}}{2\sqrt{(\Sigma_{xx} - \Sigma_{yy})^2 + 4\Sigma_{xy}}}, \quad \frac{\partial\Sigma_2}{\partial\Sigma_{xx}} = \frac{1}{2} - \frac{\Sigma_{xx} - \Sigma_{yy}}{2\sqrt{(\Sigma_{xx} - \Sigma_{yy})^2 + 4\Sigma_{xy}}},\\
&\frac{\partial\Sigma_1}{\partial\Sigma_{yy}} = \frac{1}{2} - \frac{\Sigma_{xx} - \Sigma_{yy}}{2\sqrt{(\Sigma_{xx} - \Sigma_{yy})^2 + 4\Sigma_{xy}}}, \quad \frac{\partial\Sigma_2}{\partial\Sigma_{yy}} = \frac{1}{2} + \frac{\Sigma_{xx} - \Sigma_{yy}}{2\sqrt{(\Sigma_{xx} - \Sigma_{yy})^2 + 4\Sigma_{xy}}},\\
&\frac{\partial\Sigma_1}{\partial\Sigma_{xy}} = \frac{\Sigma_{xy}}{\sqrt{(\Sigma_{xx} - \Sigma_{yy})^2 + 4\Sigma_{xy}}}, \quad \frac{\partial\Sigma_2}{\partial\Sigma_{xy}} = -\frac{\Sigma_{xy}}{\sqrt{(\Sigma_{xx} - \Sigma_{yy})^2 + 4\Sigma_{xy}}},\\
&\frac{\partial\Sigma_3}{\partial\Sigma_{zz}} = 1,\\
&\frac{\partial\Sigma_{xx}}{\partial\sigma_{xx}} = \frac{2C_{11} - C_{12} - C_{13}}{3}, \quad \frac{\partial\Sigma_{xx}}{\partial\sigma_{yy}} = \frac{2C_{12} - C_{11} - C_{13}}{3},\\
&\frac{\partial\Sigma_{yy}}{\partial\sigma_{xx}} = \frac{2C_{12} - C_{22} - C_{23}}{3}, \quad \frac{\partial\Sigma_{yy}}{\partial\sigma_{yy}} = \frac{2C_{22} - C_{12} - C_{23}}{3},\\
&\frac{\partial\Sigma_{zz}}{\partial\sigma_{xx}} = \frac{2C_{13} - C_{23} - C_{33}}{3}, \quad \frac{\partial\Sigma_{zz}}{\partial\sigma_{yy}} = \frac{2C_{23} - C_{13} - C_{33}}{3},\\
&\frac{\partial\Sigma_{xy}}{\partial\sigma_{xy}} = C_{66}.
\end{aligned} \tag{A.3}$$

Relations similar to (A.3) express each term as a function of the corresponding primed variable.

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